B.Sc. (Hons.) Part III 7^{th} Paper

Diff. Equs (continued)

Standard Form - 3

$f(p,q,z)=0.$

1: Solve $z = pq$.

Soln It is of the form $f(p,q,z)=0$.

Let $u = x+ay \Rightarrow \frac{\partial u}{\partial x}=1, \frac{\partial u}{\partial y}=a.$

$$\Rightarrow p = \frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x} = \frac{\partial z}{\partial u} = \frac{dz}{du}.$$

$$\text{and } q = \frac{\partial z}{\partial y} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y} = \frac{\partial z}{\partial u}\cdot a = a\frac{dz}{du}$$

Putting the values of p and q in the given equation, we get

$$z = \frac{dz}{du}\cdot a\frac{dz}{du}$$

$$\Rightarrow z = a\left(\frac{dz}{du}\right)^2 \Rightarrow \frac{dz}{du} = \pm\sqrt{\frac{z}{a}}$$

$$\Rightarrow \sqrt{a}\frac{dz}{\sqrt{z}} = \pm du$$

integrating, we get

$$\Rightarrow \sqrt{a}\cdot 2\sqrt{z} = \pm(u+b)$$

$$\Rightarrow 2\sqrt{az} = \pm(u+b)$$

$$\Rightarrow 4az = (u+b)^2$$

$$\Rightarrow 4az = (x+ay+b)^2$$ This is the complete integral. (1)

from (1) $4az = (x+ay+b)^2$

Differentiating (1) partially w.r.to a, we get

$4z = 2(x+ay+b).y$

Differentiating (1) partially w.r.to b, we get

$\Rightarrow 0 = 2(x+ay+b) \Rightarrow x+ay+b = 0$

Putting this value in (1), we get

$4az = 0$

$\Rightarrow z = 0$

This is the singular integral.

[$z=0 \Rightarrow \frac{\partial z}{\partial x} = 0, \ \frac{\partial z}{\partial y} = 0$ i.e $p=0, q=0$

Q. Solve $p(1+q^2) = q(z-\alpha)$

Soln The given equation is of the form $f(p,q,z)=0$.

Put $u = x+ay \Rightarrow \frac{\partial u}{\partial x} = 1,\ \frac{\partial u}{\partial y} = a.$

$$\therefore p = \frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x} = \frac{\partial z}{\partial u} = \frac{dz}{du}$$

$$q = \frac{\partial z}{\partial y} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y} = a\frac{\partial z}{\partial u} = a\frac{dz}{du}.$$

Putting these values of p and q in the given equation, we get

$$\frac{dz}{du}\left[1+a^2\left(\frac{dz}{du}\right)^2\right] = a\frac{dz}{du}(z-\alpha)$$

$$\Rightarrow \frac{dz}{du}\left[1+a^2\left(\frac{dz}{du}\right)^2 - a(z-\alpha)\right]=0$$

$$\Rightarrow \frac{dz}{du}=0 \quad \Rightarrow z = \text{constant}$$

or $$1+a^2\left(\frac{dz}{du}\right)^2 - a(z-\alpha)=0$$

$$\Rightarrow a^2\left(\frac{dz}{du}\right)^2 = a(z-\alpha)-1$$

$$\Rightarrow \frac{dz}{du} = \pm\frac{1}{a}\sqrt{a(z-\alpha)-1}$$

$$\Rightarrow \frac{a\,dz}{\sqrt{a(z-\alpha)-1}} = \pm\, du$$

integrating, we get

$$\Rightarrow 2\sqrt{a(z-\alpha)-1} = \pm\, u + b$$

$$\Rightarrow 4\left[a(z-\alpha)-1\right] = (u+b)^2$$

$$\Rightarrow 4\left[a(z-\alpha)-1\right] = (x+ay+b)^2 \quad \text{——— (1)}$$

This is the complete integral.

Differentiating (1) partially w.r.to a, we get

$$4(z-\alpha) = 2(x+ay+b)\times y \quad \text{——— (2)}$$

Diff. (1) partially w.r.to b, we get

$$0 = 2(x+ay+b) \quad \text{——— (3)}$$

From (1), (2) & (3), we have

$$z-\alpha = 0$$

This is the singular integral become

$p = \frac{\partial z}{\partial x} = 0$, $q = \frac{\partial z}{\partial y} = 0$ satisfy the given equation.